



**FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT**

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v) Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) **Use of Calculator is allowed.**

SECTION-A

- Q. 1.** (a) Let H, K be subgroups of a group G . Prove that HK is a subgroup of G if and only if $HK=KH$. (10)
- (b) If N, M are normal subgroups of a group G , prove that (10) (20)
 $NM/M \cong N/N \cap M$.
- Q. 2.** (a) If R is a commutative ring with unit element and M is an ideal of R then show that M is a maximal ideal of R if and only if R/M is a field. (10)
- (b) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then show that we can find elements a and b in F such that (10) (20)
 $1 + \alpha a^2 + \beta b^2 = 0$.
- Q. 3.** (a) Let V be a finite-dimensional vector space over a field F and W be a subspace of V . Then show that W is finite-dimensional, (10)
 $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$.
- (b) Suppose V is a finite-dimensional vector space over a field F . Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. (10) (20)

SECTION-B

- Q. 4.** (a) Use the Mean-Value Theorem to show that if f is differentiable on an interval I , and if $|f'(x)| \leq M$ for all values of x in I , then (10)
 $|f(x) - f(y)| \leq M|x - y|$
for all values of x and y in I . Use this result to show further that
 $|\sin x - \sin y| \leq |x - y|$.
- (b) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at t , and if (10) (20)
 $z = f(x, y)$ is differentiable at the point $(x, y) = (x(t), y(t))$, then
 $z = f(x(t), y(t))$ is differentiable at t and
$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y) .
- Q. 5.** (a) Evaluate the double integral (10)
$$\iint_R (3x - 2y) dx dy$$
- (b) Where R is a region enclosed by the circle $x^2 + y^2 = 1$. (10) (20)
Find the area of the region enclosed by the curves
 $y = \sin x, \quad y = \cos x, \quad x = 0, \quad x = 2\pi$.

PURE MATHEMATICS

- Q. 6.** (a) Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)
- (b) Show that if a, b and c are nonzero, then the plane whose intercepts with the coordinate axes are $x = a, y = b$, and $z = c$ is given by the equation. (10) (20)
- $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

SECTION-C

- Q. 7.** (a) Prove that a necessary and sufficient condition that $w = f(z) = u(x, y) + iv(x, y)$ be analytic in a region R is that the Cauchy-Riemann equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfied in R where it is supposed that these partial derivatives are continuous in R . (10)
- (b) Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane Z . (10) (20)
- Q. 8.** (a) Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region R . Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

- (b) Show that $\int_0^{2\pi} \frac{d\theta}{(5-3 \sin \theta)^2} = \frac{5\pi}{32}.$ (10) (20)
